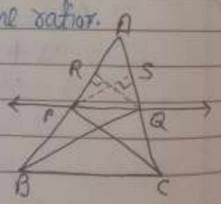
Chapter - 6 :- Triangles

- Congruent figure: The figure have same shape & size are called congruent figure.

- Similar figure: Two figure having same shape but different size are called similar figure.

- Similar Triangles: Two triangles are said to similar triangles if their corresponding angles are equal & corresponding sides are proportional.

- Basic Proportionality Theorem (B.P.T)
If a line drawn ∥ to one side of Δ intersecting other two sides at two distinct point then the other two sides are divided in same ratio.



→ Given that:
In ΔABC, $PQ \parallel BC$
To prove: $\frac{AP}{PB} = \frac{AQ}{QC}$

Const: i) Join B to Q & P to C.
ii) Draw $QR \perp AB$ & $PS \perp AC$

Proof:

Area of $\Delta\, APQ = \frac{1}{2} \times AP \times QR$ —— (i)
Area of $\Delta\, BPQ = \frac{1}{2} \times BP \times QR$ —— (ii)
Area of $\Delta\, APQ = \frac{1}{2} \times AQ \times PS$ —— (iii)
Area of $\Delta\, PQC = \frac{1}{2} \times QC \times PS$ —— (iv)

(i) ÷ (ii)
$\frac{ar(APQ)}{ar(BPQ)} = \frac{AP}{PB}$ —— v

(iii) ÷ (iv) $\left\{ \frac{ar(APQ)}{ar(PQC)} = \frac{AQ}{QC} \right.$
$\frac{ar(APQ)}{ar(BPQ)} = \frac{AQ}{QC}$ —— (vi)

{∵ ΔBPQ & ΔPQC are lie on the same base PQ and b/w same ∥ PQ ∥ BC
∴ ar(BPQ) = ar(PQC)}

from (v) & (vi)
$\frac{AP}{PB} = \frac{AQ}{QC}$ Hence proved.

Exercise 6.2

1. i) Given in ΔABC
$DE \parallel BC$
$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$
$\frac{1.5}{3} = \frac{1}{EC}$
$EC = 2cm$

ii) ∵ $DE \parallel BC$
$\frac{AD}{DB} = \frac{AE}{EC}$
$\frac{AD}{7.2} = \frac{1.8}{5.4}$
$\frac{AD}{7.2} = \frac{18}{54}$
$AD = 2.4cm$

2. i) $PE = 3.9cm$, $EQ = 3cm$
$PF = 3.6cm$, $FR = 2.4cm$

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$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3\text{ cm}$

$\frac{PE}{FR} = \frac{3.6}{2.4} = \frac{3}{2} = 1.5\text{cm}$

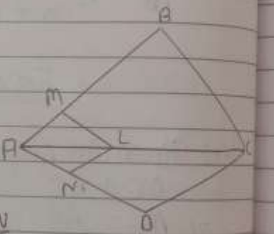
$\therefore \frac{PE}{EQ} \neq \frac{PF}{FR}$

ii) PE = 4cm, QE = 4.5cm, PF = 8cm,
~~FR = 2.4cm~~. RF = 9cm.

$\frac{\cancel{PF}}{\cancel{QF}} = \frac{PF}{PE} = \frac{8}{4} = 2\text{cm}$

$\frac{\cancel{PE}}{\cancel{RF}} = \frac{RF}{QE} = \frac{9}{4.5} = 2\text{cm}$

$\frac{PF}{PE} = \frac{RF}{QE}$



3. Given that;
ML // BC &
LN // CD

To prove; $\frac{AM}{AB} = \frac{AN}{AD}$

To proof:- In $\triangle ABC$
~~AM~~ ML // BC

$\frac{AM}{AB} = \frac{AL}{LC}$ —— (i)

In $\triangle ADC$
LN // DC

$\frac{AN}{AD} = \frac{AL}{AC}$ —— (ii)

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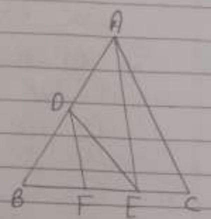
from (i) & (ii)

$\frac{AM}{AB} = \frac{AN}{AD}.$



4. Given that:
DE // AC
& DF // AE
To prove:-

$\frac{BF}{FE} = \frac{BE}{EC}$

Proof:- In $\triangle BEA$
DF // AE (by using B.P.T)

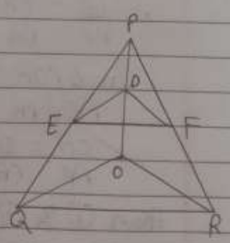
$\frac{BF}{FE} = \frac{BD}{DA}$ —— (i)

In $\triangle ABC$
$\because$ DE // AC
by using B.P.T

$\frac{BE}{EC} = \frac{BD}{DA}$ —— (ii)

From (i) & (ii)

$\frac{BF}{FE} = \frac{BE}{EC}.$



5. Given that;
DE // OQ
DF // OR
To prove; EF // QR
Proof: In $\triangle PQO$
DE // OQ (given)

$\therefore$ by using B.P.T,

$\frac{PE}{EQ} = \frac{PD}{DO}$ — (i)

In ΔPOR

$DF \parallel OR$ (given)

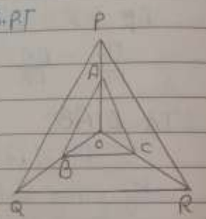
$\therefore \frac{PF}{FR} = \frac{PD}{DO}$ — (ii)

from (i) & (ii)

$\frac{PE}{EQ} = \frac{PF}{FR}$

by converse of B.P.T

$EF \parallel QR$



6. Given that;

$AB \parallel PQ$ &

$AC \parallel PR$

To show: $BC \parallel QR$

Proof:- In ΔPOQ

$AB \parallel PQ$ (given)

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ — (i) - B.P.T)

In ΔPOR

$AC \parallel PR$

$\frac{OA}{AP} = \frac{OC}{CR}$ — (ii) - B.P.T)

from (i) & (ii)

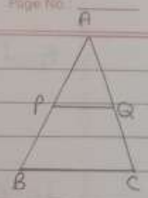
$\frac{OB}{BQ} = \frac{OC}{CR}$ (by converse of B.P.T)

$BC \parallel QR$

8. Given that;

In ΔABC

P, Q are mid points of the side AB & AC respectively.



To prove: $PQ \parallel BC$.

Proof:- P, mid point of AB (given)

$\therefore AP = PB$

$\frac{AP}{PB} = 1$ — (i)

$\because$ Q, mid point of AC (given)

$\therefore AQ = QC$

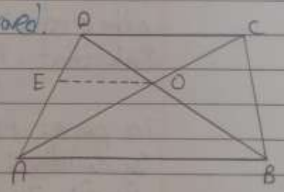
$\frac{AQ}{QC} = 1$ — (ii)

from (i) & (ii)

$\frac{AP}{PB} = \frac{AQ}{QC}$

by converse of BPT,

$\boxed{PQ \parallel BC}$ Hence, proved.



9. Given that;

In a trapezium ABCD,

$AB \parallel DC$.

AC & BD intersect at O.

To prove:- $\frac{AO}{BO} = \frac{CO}{DO}$

Const:- Draw $DE \parallel AB \parallel CD$.

Proof: In ΔADC,

$OE \parallel DC$

$\frac{DE}{EA} = \frac{CO}{AO}$ —— (i) ← B.P.T)

In ΔABD

$OE \parallel AB$

$\frac{DE}{EA} = \frac{DO}{BO}$ —— (ii) - {B.P.T}

from (i) & (ii)

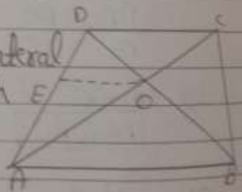
$\frac{CO}{AO} = \frac{DO}{BO}$

$(AO)\cdot(DO) = (BO)\cdot(CO)$

$\frac{AO}{BO} = \frac{CO}{DO}$

Hence proved

10. Given that;
Diagonals of quadrilateral ABCD intersect each other at O:
Such that: $\frac{AO}{BO} = \frac{CO}{DO}$



To prove: ABCD is trapezium.
Construction: Draw $OE \parallel AB$
Proof: In ΔABD

$OE \parallel AB$

$\frac{DE}{EA} = \frac{DO}{BO}$ —— (i) — B.P.T)

$\therefore \frac{AO}{BO} = \frac{CO}{DO}$ (given).

or, $\frac{DO}{BO} = \frac{CO}{AO}$ —— (ii)

from (i) & (ii)

$\frac{DE}{EA} = \frac{CO}{AO}$

by converse of B.P.T

$OE \parallel DC$ —— (iii)

$\because OE \parallel AB$ —— (iv) by construction

from (iii) & (iv)

$AB \parallel CD$.

$\therefore$ ABCD is a trapezium.

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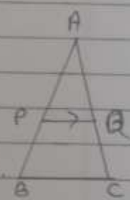

7. Given: P is mid-point of AB
& ~~Q is mid-point of AC.~~

$PQ \parallel BC$

To prove ⇒ $AQ = QC$ (or bisects AC)

Proof ⇒ $AP = PB$ (P is midpoint)

$PQ \parallel BC$ (given)

So, $\frac{AP}{PB} = \frac{AQ}{QC}$ (by B.P.T)

It can be written as

$\frac{AP}{AP} = \frac{AQ}{QC}$ (PA = PB)

$QC = AQ$

So, Q bisects AC.