Date / /

Unit-3 ⇒ Pair of Linear Equations in Two variables

* Notes:-

1. Two linear eqs in the same two variables are called a pair of linear eq in two variables. The most general form of pair of linear eq. is.

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

where, $a_1, a_2, b_1, b_2, c_1, c_2$ are real nos such that $a_1^2, b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$

2. A pair of linear equations in two variables can be represented, and solved, by the.
   i) graphical method.
   ii) algebric method.

3. Graphical method:
   The graph of a pair of linear eq. in two variables is represented by two lines.

i) If the lines intersect at a point, then that point gives the unique solution of the two eqs. In this case, the pair of equations is consistent.

ii) In the line coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equations is dependent (consistent).

iii) If the lines are parallel, then the pair of eq. has no solution. In this case, the pair of eq.

is inconsistent

4. Algebraic method : We have discussed the following methods for finding the solution(s) of a pair of linear eq.s
   i) Substitution method
   ii) Elimination "
   iii) Cross-multiplication method.

5. If a pair of linear eq.s is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then the following situation can arise:-

   i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: In this case, the pair of linear equation is consistent

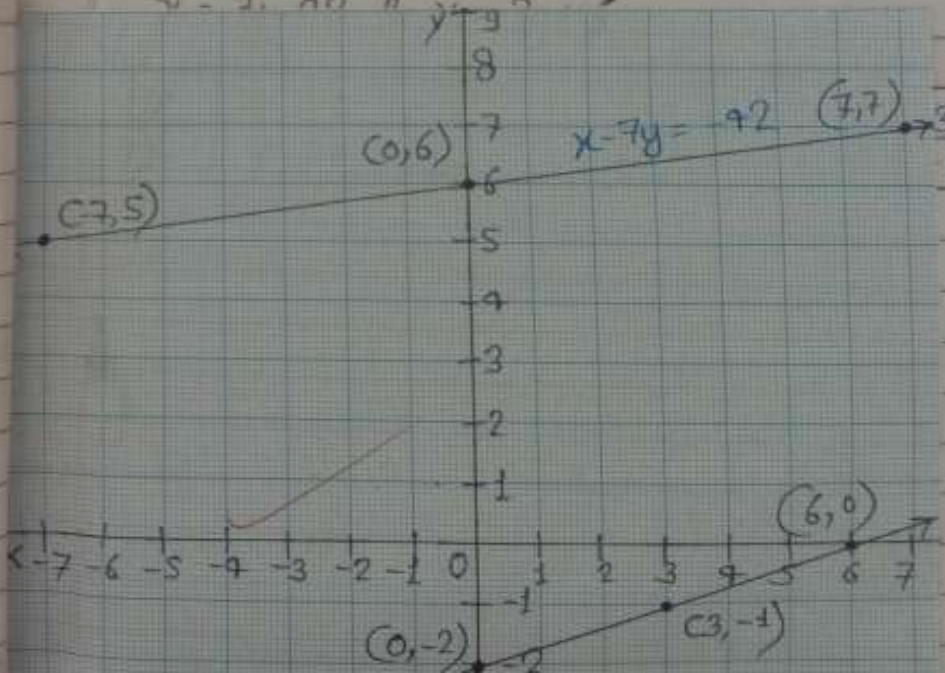
   ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$: In this case, the pair of linear eq. is inconsistent

   iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$: In this case, the pair of linear eq. is dependent and consistent.

6. There are several situations which can be mathematically represented by two eq.s that are not linear to start with. But we alter them so that they are reduced to a pair of linear equations.

Exercise-3.1

1. Let x and y be the present age of Aftab and his daughter respectively.
Given that,
seven years ago,
Aftab = 7 × his daughter
$(x-7) = 7 \times (y-7)$
$x - 7 = 7y - 49$
$x = 7y - 42$ — (i)
Aftab = 3 × (his daughter)
$(x+3) = 3(y+3)$
$x = 3y + 6$ — (ii)
algebraically representation of given situation is,



2. Let cost of one bat = $2x$ and one ball = y

ACQ,

i) $3x + 6y = 3900$

$x + 2y = 1300$

$x = 1300 - 2y$

Solutions; i) $y = 0$

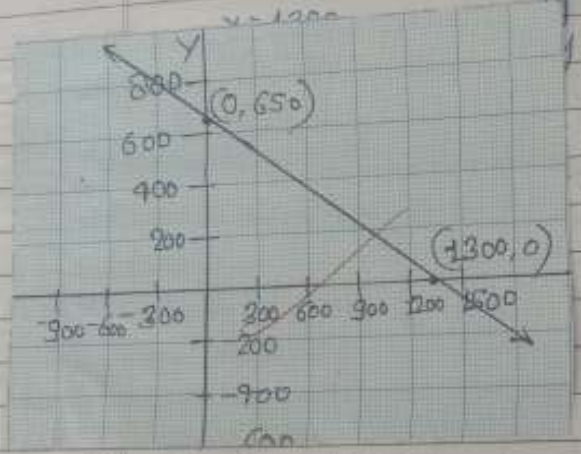
$x = 1300 - 2(0)$

ii) $x = 0$

$0 = 1300 - 2y$

$2y = 1300$

$y = 650$

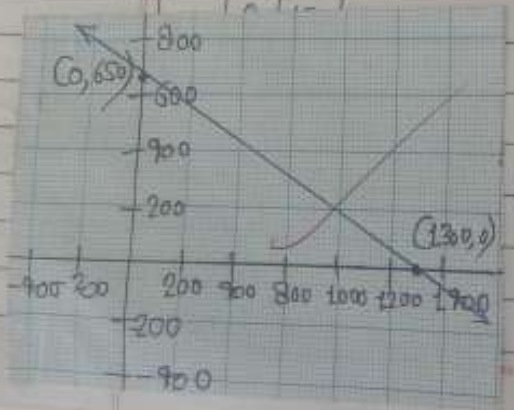


ii) $x + 2y = 1300$

Solutions; $y = 0$, $x = 0$

$x + 2(0) = 1300$ $\quad 0 + 2y = 1300$

$x = 1300$ $\quad y = 650$



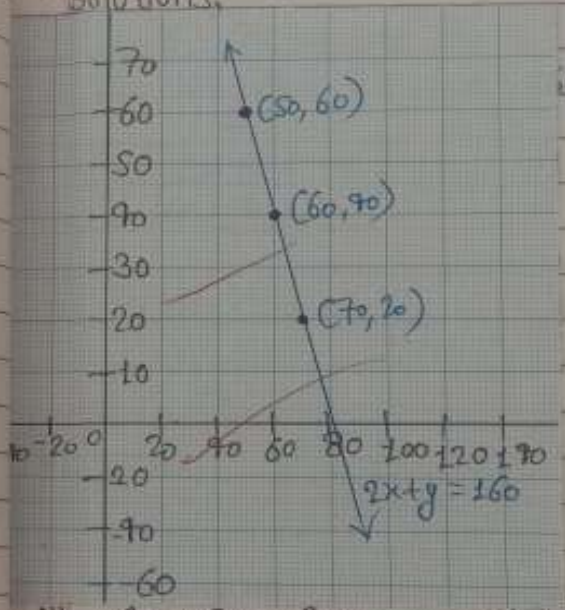
3. Let the cost of each kg of apples be x and each kg of grapes be y.

i) ACQ,

$2x + y = 160$ — (i)

$x = \frac{160 - y}{2}$

Solutions;



$\frac{60 - 70}{2} = 60$, $x = \frac{160 - 60}{2} = 50$

ii) $4x + 2y = 300$ — (ii)

$2x + y = 150$

$y = 150 - 2x$

Solutions;

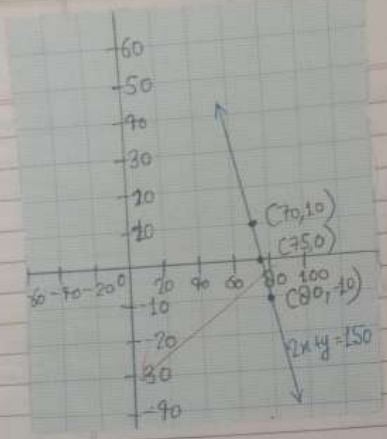
$x = 70, 75$ and 80

$y = 150 - 140 = 10$, $y = 150 - 150 = 0$, $y = 150 - 160 = -10$

x	70	75	80
y	10	0	-10

29/09/19

Page No. 92



Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ be the pair of linear equation in two variable.

If,

	Numbers of Solutions.
i) lines intersecting then, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	one unique
ii) lines are parallel then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	No solution
iii) Coincident then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Infinetly many solutions.

7. i) Let the no. of boys be x and girls be y

Page No. 93

ACQ,

$x + y = 10$ —— (i)

$x = 10 - y$

Solutions;

$y = 4, 5$ and 6.

$x = 10-4 = 6$, $x = 10-5 = 5$, $x = 10-6 = 4$

x	4	5	6
y	6	5	4

$x = y + 4$

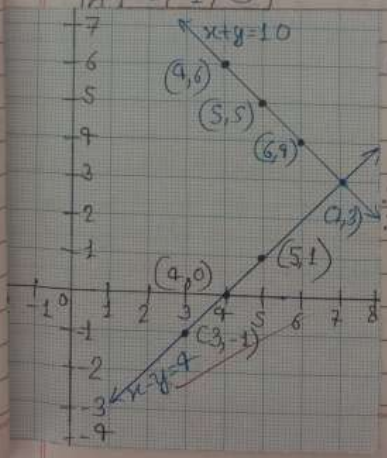
$y = x - 4$ —— (ii)

Solutions,

$x = 3, 5, 7$

$y = 3-4 = -1$, $y = 5-4 = 1$, $y = 7-4 = 3$

x	3	5	7
y	-1	1	3



• both lines intersect each other at (7,3)
=) solution = (7,3)

∴ No. of girls = x = 7
∴ " " boys = y = 3

ii) Let cost of one pencil be x and one pen be y.

ACQ,

$5x + 7y = 50$

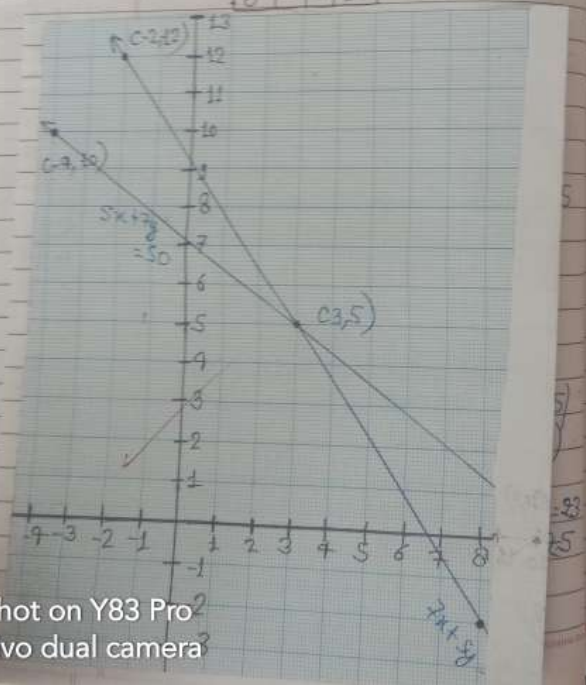
$x = \frac{10 - 7y}{5}$ ——— (i)

solutions:

$y = 0, 5, 10$

$x = 10 - 0 = 10$, $x = \frac{15}{5} = 3$, $x = \frac{-20}{5} = -4$

x	10	3	-4
y	0	5	10



Shot on Y83 Pro
vivo dual camera

Page No. 95

2. i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

$\frac{a_1}{a_2} = \frac{5}{7}$, $\frac{b_1}{b_2} = \frac{-4}{6}$, $\frac{c_1}{c_2} = \frac{8}{-9}$

$\therefore$ lines are intersecting

ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

$\frac{a_1}{a_2} = \frac{9}{18}$, $\frac{b_1}{b_2} = \frac{3}{6}$, $\frac{c_1}{c_2} = \frac{12}{24}$

$\therefore$ lines are coincident

iii) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

$\frac{a_1}{a_2} = \frac{6}{2}$, $\frac{b_1}{b_2} = \frac{-3}{-1}$, $\frac{c_1}{c_2} = \frac{10}{9}$

$\therefore$ lines are parallell.

3. i) $3x + 2y = 5$

$2x - 3y = 7$

$\Rightarrow \frac{a_1}{a_2} = \frac{3}{2}$, $\frac{b_1}{b_2} = \frac{2}{-3}$, $\frac{c_1}{c_2} = \frac{5}{7}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ Consistent

ii) $2x - 3y = 8$

$4x - 6y = 9$

$\Rightarrow \frac{a_1}{a_2} = \frac{2}{4}$, $\frac{b_1}{b_2} = \frac{3}{6}$, $\frac{c_1}{c_2} = \frac{8}{9}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$ ⇒ Inconsistent.

iii) $\frac{3}{2}x + \frac{5}{3}y = 7$

$9x - 10y = 14$

⇒ $\frac{a_1}{a_2} = \frac{3}{2} = \frac{1}{6}$, $\frac{b_1}{b_2} = \frac{5}{-10}$ ⇒ $\frac{5}{-30} = \frac{-1}{6}$

$\frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$

⇒ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ ⇒ Consistent.

iv) $\frac{4}{3}x + 2y = 8$, $2x + 3y = 12$

$\frac{a_1}{a_2} = \frac{4}{3} = \frac{2}{3}$, $\frac{b_1}{b_2} = \frac{2}{3}$, $\frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

⇒ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ⇒ Consistent.

4. i) $x + y = 5$

$2x + 2y = 10$

$\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2}$

⇒ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ⇒ Consistent.

So, $x + y = 5$

$x = 5 - y$

solutions,

putting, 1, 2, and 3

$x = 5-1$, $x = 5-2$, $x = 5-3$

$x = 4$, $x = 3$, $x = 2$

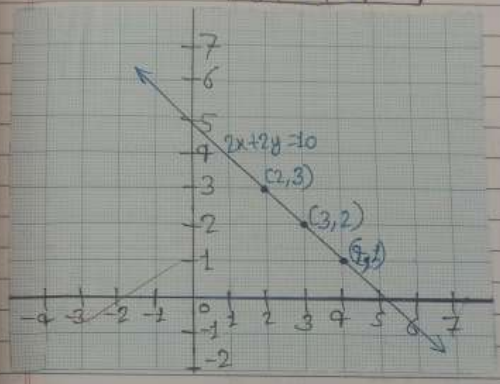
Shot on Y83 Pro
vivo dual camera

Page No. 97

x	4	3	2
y	1	2	3

∴ $x = \frac{10 - 2y}{2}$

x	4	3	2
y	1	2	3



ii) $x - y = 8$, $3x - 3y = 16$

$\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$, $\frac{c_1}{c_2} = \frac{1}{2}$

⇒ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ ⇒ Inconsistent.

5. Let the width of garden be x and length of garden be y.

Given,

$y = x + 4$ —(i)

and,

Page No. 98

Half of perimeter of garden = 36

$\frac{2(x+y)}{2} = 36$

$x + y = 36$ ... (i)

From (i) $y = x + 4$

$x + x + 4 = 36$

$2x = 32$

$x = 16m$

From (i) $x + y = 36$

$16 + y = 36$

$y = 20m$

6. Give the linear equation;

$2x + 3y - 8 = 0$

i) Intersecting lines :- $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, new equation = $2x + 4y - 6 = 0$

as,

$\frac{a_1}{a_2} = \frac{2}{2} = 1, \frac{b_1}{b_2} = \frac{3}{4}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

ii) Parallel lines :- $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, new line can be $4x + 6y - 8 = 0$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-8} = 1$

So, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

iii) Coincident lines $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Page No. 99

So,

new equation = $6x + 9y - 27 = 0$

$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$

as,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

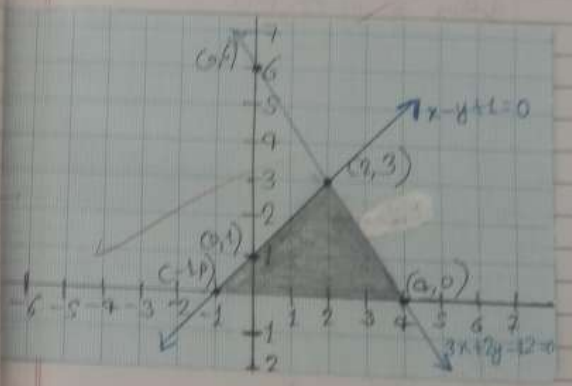
7. eq (i) = $x - y + 1 = 0$, eq (ii) = $3x + 2y - 12 = 0$

as, $x = y - 1$

as, $x = \frac{12 - 2y}{3}$

x	0	1	2	-1
y	1	2	3	0

x	a	2	0
y	0	3	6



Date 29 07 2019 Page No. 50

Exercise 3.3

1. i) $x+y=17$, $x-y=7$

$\Rightarrow$ $x+y=17$ —— (i)

$x-y=7$ —— (ii)

from eq. (i)

$x=17-y$ —— (iii)

putting value in eq. (ii)

$(17-y)-y=7$

$17-2y=7$

$2y=10$

$y=5$

Putting y in eq. (iii)

$x=17-5=9$

$\Rightarrow x=9,\ y=5$

v) $\sqrt{2}x+\sqrt{3}y=0$ —— (i)

$\sqrt{3}x-\sqrt{8}y=0$ —— (ii)

from eq. (i)

$\sqrt{2}x=-\sqrt{3}y$

$x=\frac{-\sqrt{3}y}{\sqrt{2}}$ —— (iii)

Putting value in eq. (ii)

$\sqrt{3}\left(\frac{-\sqrt{3}y}{\sqrt{2}}\right)-\sqrt{8}y=0$

$\frac{-3y}{\sqrt{2}}-2\sqrt{2}y=0$ $(\sqrt{8}=2\sqrt{2})$

$y\cdot\left(\frac{-3}{\sqrt{2}}-2\sqrt{2}\right)=0$

$y=0$

Date Page No. 51

putting 'y' in (iii) Eq)

$x=0$

Hence, $x=0,\ y=0$

vi) $\frac{3x}{2}-\frac{5y}{2}=-2$ —— (i)

$\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$ —— (ii)

from eq. (i)

$\frac{3\times 3x-2\times 5y}{6}=-2$

$9x-10y=-12$

$x=\frac{-12+10y}{9}$

$\Rightarrow$ Putting value in eq. —— (ii)

$\Rightarrow$ $\frac{\frac{-12+10y}{9}}{3}+\frac{y}{2}=\frac{13}{6}$

$\frac{-12+10y}{27}+\frac{y}{2}=\frac{13}{6}$

$\frac{-24+20y+27y}{54}=\frac{13}{6}$

$47y=117+24$

$47y=141$ $\quad y=\frac{141}{47}=3$

$\Rightarrow$ putting y in (iii)

$x=\frac{-12+10y}{9}=\frac{-12+10\times 3}{9}=\frac{18}{9}=2$

$y=3$

$x=2$

Page No. 52

2. To solve,
$2x + 3y = 11$ — (i)
$2x - 9y = -29$ — (ii)
From eq. (i)
$2x = 11 - 3y$ — (iii)
putting values in eq. (ii)
$(11 - 3y) - 9y = -29$
$11 - 12y = -29$
$-12y = -35$
$y = 5$
Putting values in eq. (iii)
$2x = 11 - 3y$
$2x = 11 - 3 \times 5$
$2x = 11 - 15$
$2x = -4$
$x = -2$
Now putting values in eq.
$y = mx + 3$
$5 = m \times -2 + 3$
$-2m = 2$
$m = -2 = -1$
⇒ value of m is -1.

3. i) Let larger no. be x and smaller no. be y.
ACQ,
$x - y = 26$
$x = 26 - y$ — (i)

Page No. 53

Putting values in eq. (ii)
$x = 3y$
$26 - y = 3y$
$-2y = -26$
$y = 13$
So, $x = 3y = 39$
⇒ The nos are 13 and 39.

ii) Let the first angle be x and second be y.
ACQ, sum of
$x + y = 180$ (Supplementary angles = 180°)
$x = 180 - y$ — (i)
and,
$x - y = 18$ — (ii) (given)
∴ putting value of x in eq. (ii)
$180 - y - y = 18$
$-2y = 18 - 180$
$y = \frac{-162}{-2} = 81°$
So, $x = 180 - y = 180 - 81 = 99°$
⇒ The two angles are 81° and 99°

iii) Let cost of each bat be x and each ball be y.
ACQ,
$7x + 6y = 3800$ — (given)
$6y = 3800 - 7x$
$y = \frac{3800 - 7x}{6}$ — (i)
and.

$3x + 5y = 1750$ —(ii) (given)

∴ putting value of y

$3x + 5\times\frac{3800-7x}{6} = 1750$

$18x + 19000 - 35x = 1750\times 6$

$-17x = 10500 - 19000$

$-17x = -8500$

$x = 500$

⇒ putting value of x in eq (i)

$y = \frac{3800 - 7\times 500}{6} = \frac{300}{6} = 50$

⇒ cost of each bat ⇒ ₹500

" " " ball = ₹50

iv) Let fixed cost for taxis = ₹x

and variable cost per km = ₹y

We know,

Total cost = Fixed charge + variable charge

A/Q,

$x + 10y = 105$ (given)

$x = 105 - 10y$ —(i)

⇒ for journey 15km charge paid = ₹155

$x + 15y = 155$ —(ii)

putting values of eq. (i) in (ii)

$105 - 10y + 15y = 155$

$5y = 155 - 105 = 50$

$y = 10$.

for, 'x' ⇒ $105 - 10y$

⇒ $105 - 10\times 10$

⇒ $105 - 100$

So, people have to pay for travelling a distance of 25km

$x + 25y = 5 + 25\times 10$ ⇒ ₹255 Ans.

v) Let numerator be x and denominator be y

∴ fraction = $\frac{x}{y}$

A/Q,

$\frac{9}{11}$, if 2 is added in numerator and denominator

⇒ $\frac{x+2}{y+2} = \frac{9}{11}$

⇒ $11x + 22 = 9y + 18$

$11x = 9y - 4$

$x = \frac{9y-4}{11}$ —(i)

∴ and if 3 is added to both numerator and denominator it become $\frac{5}{6}$

⇒ $\frac{x+3}{y+3} = \frac{5}{6}$

$6x + 18 = 5y + 15$ —(ii)

putting values of (i) in (ii)

$6\times\frac{9y-4}{11} + 18 = 5y + 15$

$54y - 24 = 55y - 33$

$-y = -9$

$y = 9$

∴ putting value of y in eq (i)

$x = \frac{9\times 9 - 4}{11} = \frac{77}{11} = 7$

The fraction is ⇒ $\frac{7}{9}$ Ans.

Page No. 56

vi) Let present age of Jacob be x and present age of his son $= y$.

$\Rightarrow$ ACQ,

$\therefore$ 5 years hence,

Age of Jacob $= x+5$

Age of son $= y+5$

and age of Jacob is 3 times of his son. — (given)

$x+5 = 3(y+5)$

$x+5 = 3y+15$

$x = 3y+10$ —— (i)

$\therefore$ 5 years ago,

Jacob $= x-5$

Son $= y-5$

and jacob age was 7 times of his son.

$\Rightarrow x-5 = 7(y-5)$

$x-5 = 7y-35$

$x-7y = -30$ —— (ii)

putting x in (ii)

$3y+10-7y = -30$

$-4y = -40$

$y = 10$

Putting value of y in (i)

$x = 3\times10+10$

$x = 30+10 = 40$

Ans $\Rightarrow$ present age of Jacob $= 40$ years and of his son $= 10$ years.

Shot on Y83 Pro
vivo dual camera

Date 21.07.2019 Page No. 57

Exercise 3.4

1. i) $x+y = 5$

$2x-3y = 4$

$\Rightarrow$ by elimination method:

$x+y = 5$ —— (i)

$2x-3y = 4$ —— (ii)

$\Rightarrow$ multiply eq. (i) by '2',

$2x+2y = 10$ —— (iii)

$\therefore$ subtract eq. (ii) from (iii)

$2x+2y = 10$

$2x-3y = 4$

$+5y = 6$

$y = \frac{6}{5}$

$\Rightarrow$ putting value in eq. (i)

$x = 5-y$

$x = 5-\frac{6}{5}$

$x = \frac{19}{5}$

$\star$ by substitution method:

$x+y = 5$ —— (i)

$2x-3y = 4$ —— (ii)

from eq. (i)

$x = 5-y$ (iii)

putting values of (iii) in (ii)

$2(5-y)-3y = 4$

$10-2y-3y = 4$

$-5y = -6$

putting value of y in (iii)

$x = \frac{5-6}{5} = \frac{-1}{5}$

iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

* by elimination method:-

$\frac{x}{2} + \frac{2y}{3} = -1$ —— (i)

$x - \frac{y}{3} = 3$ —— (ii)

multiplying eq. (i) by 2, we get

$x + \frac{4y}{3} = -2$ —— (iii)

Subtracting eq. (ii) from (iii)

$x + \frac{4y}{3} = -2$

$x - \frac{y}{3} = 3$

$\frac{5y}{3} = -5$

$y = -5 \times 3 = -3$

putting value of y in eq. (ii)

$x + \frac{4y}{3} = -2$

$x + \frac{4 \times -3}{3} = 3$

$x - 4 = -2$

$x = 2$

* by substitution method:-

$\frac{x}{2} + \frac{2y}{3} = -1$ —— (i)

$x - \frac{y}{3} = 3$ —— (ii)

(iii)

(iii)

putting value in eq. (i)

$\frac{3+y}{6} + \frac{2y}{3} = -1$

$\frac{3+y+4y}{6} = -1$

$5y = -6-3 \Rightarrow -15$

$y = \frac{-15}{5} = -3$

Putting value of y in eq. (iii)

$x = \frac{3+y}{3} \Rightarrow x = \frac{3+-3}{3}$

$\Rightarrow 3x = 3-3 \Rightarrow 3x = 6$

$x = 2$

2. i) Let the fraction is $= \frac{x}{y}$

According condition (i)

$\Rightarrow \frac{x+1}{y+1} = 1$

$x+1 = y-1$

$x-y = -2$ —— (i)

According condition (ii)

$\frac{x}{y+1} = \frac{1}{2}$

$2x = y+1$

$2x-y = 1$ —— (ii)

subtracting (i) from (ii)

$2x - y = 1$

$x - y = -2$

$x = 3$

putting value in eq. (i)

$3 - y = -2$

$y = -5$

$y = 5$

∴ the required fraction is $\frac{3}{5}$.

ii) Let present age of Nuri be x and Sonu be y

→ According to condition (i)

five years before Nuri's age is 3 times that of Sonu's age.

$(x-5) = 3(y-5)$

$x - 3y = -10$ ——— (i)

→ According to condition (ii)

$x+10 = 2(y+10)$

$x - 2y = 10$ ——— (ii)

⇒ Subtracting (i) from (ii)

$x - 2y = 10$

$x - 3y = -10$

$y = 20$

putting value in eq. (i)

$x - 60 = -10$

$x = 50$

∴ Present age of Nuri = 50 years

" " Sonu = 20 years

iii) Let the unit digit of no. = x.

Ten's digit = y

So, number = $10y + x$

∴ No. formed after reversing the digits = $10x + y$

ACQ,

$x + y = 9$ ——— (i)

$9(10y + x) = 2(10x + y)$

$= 90y + 9x = 20x + 2y$

$\Rightarrow 88y - 11x = 0$

$\Rightarrow -x + 8y = 0$ ——— (ii)

adding eq. (i) and ii)

$x + y = 9$

$-x + 8y = 0$

$9y = 9$

$y = 1$

putting value in eq. (i)

$x + 1 = 9$

$x = 8$

The required no; $10y + x = 10 + 1 + 8 = 18$.

iv) Let the no. of ₹50 notes be x and ₹100 notes be y.

ACQ,

$x + y = 25$ ——— (i)

$50x + 100y = 2000$ ——— (ii)

multiply eq. (i) by 50.

$50x + 50y = 1250$ ——— (iii)

Subtracting (iii) from (ii)

$50x + 100y = 2000$

$50x + 50y = 1250$

$50y = 750$, $y = 15$

Page No. 62

putting value in (i)

$x+y=25$

$15+y=25$

$y=10$

⇒ Meena has 10 notes of ₹50 and 15 notes of ₹100.

v) Let the fixed charge for first three days $= x$

Let each day charge there after be y

ATQ,

$x+2y=27$ ——(i)

$x+2y=21$ ——(ii)

Subtracting (ii) from (i)

$x+2y=27$

$x+2y=21$

$2y=6$

$y=3$

putting value in eq. (i)

$x+12=27$

$x=15$

fixed charge = ₹15

Charge per day = ₹3

Date 30/09/19

Page No.

Exercise 3.5

1. i) $x-3y-3=0$

$3x-9y-2=0$

$\frac{a_1}{a_2}=\frac{1}{3}$, $\frac{b_1}{b_2}=\frac{-3}{-9}$, $\frac{c_1}{c_2}=\frac{-3}{-2}$

$\Rightarrow \frac{1}{3}\neq\frac{-3}{9}=\frac{-3}{2}$

∴ lines are parallel. It has no solution.

ii) $2x+y-5=0$

$3x+2y-8=0$

$\frac{a_1}{a_2}=\frac{2}{3}$, $\frac{b_1}{b_2}=\frac{1}{2}$, $\frac{c_1}{c_2}=\frac{-5}{-8}$

$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}=\frac{c_1}{c_2}$

∴ lines are intersect each other, it has a unique solution.

$\frac{x}{\frac{1}{2}\ \ -5}=\frac{y}{-5\ \ 2}=\frac{1}{\frac{2}{3}\ \ \frac{1}{2}}$

$\frac{x}{-8+10}=\frac{y}{-15+16}=\frac{1}{4-3}$

$\Rightarrow \frac{x}{2}=\frac{y}{1}=1$

$\rightarrow \frac{x}{2}=1 \Rightarrow x=2$

$\rightarrow \frac{y}{1}=1 \Rightarrow y=1$

iii) $3x - 5y - 20 = 0$

$6x - 10y - 90 = 0$

$\frac{a_1}{a_2} = \frac{3}{6}$, $\frac{b_1}{b_2} = \frac{-5}{-10}$, $\frac{c_1}{c_2} = \frac{-20}{-90}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ The lines are coincident to each other so, it has many solutions.

iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{-3}{-3}$, $\frac{c_1}{c_2} = \frac{-7}{15}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ lines are intersecting so, it has unique solution

$\Rightarrow \frac{x}{-3 \quad -7} = \frac{y}{-7 \quad 1} = \frac{1}{\frac{1}{3} \quad -3}$

$\frac{-3 \quad -15}{-15 \quad \frac{1}{3}} \quad \frac{\frac{1}{3} \quad -3}{}$

$\Rightarrow \frac{x}{+5 - 21} = \frac{y}{-21+15} = \frac{1}{-3+9}$

$\Rightarrow \frac{x}{29} = \frac{y}{-6} = \frac{1}{6}$

$\Rightarrow \frac{x}{29} = \frac{1}{6} \Rightarrow x = 4$

$\Rightarrow \frac{y}{-6} = \frac{1}{6} \Rightarrow y = -1$

2. i) $2x + 3y = 7$

$(a-b)x + (a+b)y - (3a+b-2) = 0$

∴ a pair of linear equations (say $a_1x + b_1x + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$) have infinite solutions,

if, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ from the given equations:-

$\frac{a_1}{a_2} = \frac{2}{a-b}$, $\frac{b_1}{b_2} = \frac{3}{a+b}$, $\frac{c_1}{c_2} = \frac{7}{(3a+b-2)}$

for infinitely many solutions,

$\frac{2}{a-b} = \frac{7}{3a+b-2}$

$6a + 2b - 4 = 7a - 7b$

$a - 9b - 4$ ——— (i)

$\frac{2}{a-b} = \frac{3}{a+b}$

$2a + 2b = 3a - 3b$

$a - 5b = 0$ ——— (ii)

Subtract (i) from (ii)

$a - 5b = 0$

$a - 9b = -4$

$- \quad + \quad +$

$4b = 4$

$b = 1$

Putting the value in eq. (i)

$a - 5 \times 1 = 0$

$a = 5$,

ii) $3x + y - 1 = 0$
$(2k-1)x + (k-1)y - 2k - 1 = 0$

$\frac{a_1}{a_2} = \frac{3}{2k-1}$, $\frac{b_1}{b_2} = \frac{1}{k-1}$, $\frac{c_1}{c_2} = \frac{-1}{2k+1}$

for no solution,
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow$ So,
$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$

$\frac{3}{2k-1} = \frac{1}{k-1}$

$3k - 3 = 2k - 1$

$k = 2$

$\therefore$ for, $k = 2$, the given eq. has no solution

3. $3x + 5y = 9$ — (i) (Substitutional)
$3x + 2y = 7$ — (ii)

$\Rightarrow$ from eq. (ii)
$x = \frac{7 - 2y}{3}$ — (iii)

putting this value in eq. (i)
$3\left(\frac{7-2y}{3}\right) + 5y = 9$
$32y - 10y + 15y = 27$
$-y = -5$
$y = 5$

putting in (ii)

$3x + 10 = 7$
$3x = -6$
$x = -2$
$\Rightarrow x = -2, y = 5$

$\Rightarrow$ by cross multiplication:-
$3x + 5y - 9 = 0$
$3x + 2y - 7 = 0$ $\Big/ \frac{a_1}{a_2} = \frac{3}{3}$, $\frac{b_1}{b_2} = \frac{5}{2}$, $\frac{c_1}{c_2} = \frac{-9}{-7}$

$\Rightarrow$
$\frac{x}{5 \quad -9} = \frac{y}{-9 \quad 3} = \frac{1}{3 \quad 5}$
$\frac{}{2 \quad -7} \quad \frac{}{-7 \quad 3} \quad \frac{}{3 \quad 2}$

$\Rightarrow \frac{x}{-20 + 18} = \frac{y}{-27 + 32} = \frac{1}{10 - 15}$

$\Rightarrow \frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$

$\Rightarrow \frac{x}{-2} = \frac{1}{1}$, $x = -2$

$\Rightarrow \frac{y}{5} = \frac{1}{1} \Rightarrow y = 5$

4. i) Let x be the fixed charge of the food and y be the charge of food per day.
ACQ
$x + 20y = 1000$ — (i)
$x + 28y = 1180$ — (ii)

subtract (i) from (ii)

$x + 2y = 1180$
$x + 20y = 1000$
$- \quad - \quad -$
$6y = 180$ — (iii)
$y = 30$
putting 'y' in eq. (i)
$x + 20 \times 30 = 1000$
$x = 1000 - 600 = 400$
⇒ fixed charge = ₹400
charge per day = ₹30

ii) Let the fraction be $\frac{x}{y}$
ACQ,
$\frac{x-1}{y} = \frac{1}{3} \Rightarrow 3x - y = 3$ — (i)
$\frac{x}{y+8} = \frac{1}{4} \Rightarrow 4x - y = 8$ — (ii)
Subtracting (i) from (ii)
$4x - y = 8$
$3x - y = 3$
$- \quad + \quad -$
$x = 5$ — (iii)
⇒ putting 'x' in eq. (i)
$15 - y = 3$
$y = 12$
⇒ the fraction = $\frac{5}{12}$

iii) Let the number of right answer and wrong answer be x and y respectively.
ACQ,
$3x - y = 70$ — (i)
$4x - 2y = 50$
$2x - y = 25$ — (ii)
subtracting (ii) from (i)
$3x - y = 70$
$2x - y = 25$
$- \quad + \quad -$
$x = 15$ — (iii)
putting x in eq. (ii)
$30 - y = 25$
$y = 5$
∴ right answer = 15
wrong " = 5
Total question = 20

iv) Let the speed of 1st car and 2nd car be u km/h and v km/h respectively.
∴ Respective speed of both cars while they are travelling in same direction = (u-v)
∴ travelling towards each other = (u+v)
ACQ,
$5(u - v) = 100$
$u - v = 20$ — (i)
$1(u + v) = 100$ — (ii)

adding both the eq.

$u - v = 20$

$u + v = 100$

$2u = 120$

$u = 60$ Km/h — (iii)

putting 'u' in eq (ii)

$u + v = 100$

$60 + v = 100$

$v = 40$ Km/h.

v) Let length be x and breadth be y

$\therefore$ Area $= xy$

ACQ,

$(x-5)(y-3) = xy - 9$

$3x - 5y - 6 = 0$ — (i)

$(x+3)(y+2) = xy + 67$

$2x + 3y - 61 = 0$ — (ii)

by cross multiplication method;

$\frac{a_1}{a_2} = \frac{2}{3}$, $\frac{b_1}{b_2} = \frac{-5}{3}$, $\frac{c_1}{c_2} = \frac{-6}{-61}$

$\frac{x}{-5 \quad -6} = \frac{y}{-6 \quad 2} = \frac{1}{2 \quad -5}$

$\frac{x}{3 \quad -61} = \frac{y}{-61 \quad 3} = \frac{1}{3 \quad 3}$

$\frac{x}{305 - 18} = \frac{y}{-12 + 183} = \frac{1}{9 + 10}$

$\Rightarrow \frac{x}{323} = \frac{y}{171} = \frac{1}{19}$

$x = 19$

$y = 19$

Exercise - 3.6

1. ii) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

$\Rightarrow$ Putting $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$ in the given equation.

$2p + 3q = 2$ — (i)

$4p - 9q = -1$ — (ii)

multiply eq. (i) by 3

$6p + 9q = 6$ — (iii)

Adding eq (ii) and (iii)

$10p = 5$

$p = \frac{1}{2}$ — (iv)

Putting in eq (i)

$\Rightarrow 2 \times \frac{1}{2} + 3q = 2$

$3q = 1$

$q = \frac{1}{3}$

$p = \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$

$q = \frac{1}{\sqrt{y}} = \frac{1}{3} = \sqrt{y} = 3, \Rightarrow y = 9$

vi) $6x + 3y = 6xy$ — (i)
$2x + 4y = 5xy$ — (ii)

dividing eq. (i) by xy: $\frac{6x}{xy} + \frac{3y}{xy} = \frac{6xy}{xy}$

$\frac{6}{y} + \frac{3}{x} = 6$ ---- (i)

dividing eq. (ii) by xy: $\frac{2x}{xy} + \frac{4y}{xy} = \frac{5xy}{xy}$

$\frac{2}{y} + \frac{4}{x} = 5$ ---- (ii)

Putting $\frac{1}{x} = p$ and $\frac{1}{y} = q$. we get,
$6q + 3p - 6 = 0$
$2q + 4p - 5 = 0$
by cross multiplication method, we get,

$\frac{p}{-30-(-12)} = \frac{q}{-24-(-15)} = \frac{1}{6-24}$

$\Rightarrow \frac{p}{-18} = \frac{q}{-9} = \frac{1}{-18}$

After comparing we get,
$p = 1$ and $q = \frac{1}{2}$
now,
$p = \frac{1}{x} = 1$ and $q = \frac{1}{y} = \frac{1}{2}$

Hence, $x = 1$ and $y = 2$

vii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

$\frac{1}{3x+y} = p$ and $\frac{1}{3x-y} = q$, we get,

$p + q = \frac{3}{4}$ ---- (i)

$\frac{p}{2} - \frac{q}{2} = \frac{-1}{8}$

$p - q = \frac{-1}{4}$ ---- (ii)

Adding (i) and (ii),
$2p = \frac{3}{4} - \frac{1}{4} \Rightarrow \frac{2}{4} = \frac{1}{2}$

$\therefore p = \frac{1}{4}$

Putting value of p in (i)
$\Rightarrow \frac{1}{4} - q = \frac{-1}{4}$

$q = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Now,
$p = \frac{1}{3x+y}$ so, $3x + y = 4$ ---- (iii)

$q = \frac{1}{3x-y}$ $\Rightarrow$ $3x - y = 2$ ------ (iv)

Adding eq. (iii) and (iv), we get.

$6x = 6$

$x = 1$

Putting value of x in eq. (iii)

$3(1) + y = 4$

$y = 1$

$x = 1, y = 1$

2. i) Let the speed of Ritu in still water and the speed of stream be x km/h and y km/h respe...

The speed of Ritu while rowing:-

upstream = $(x - y)$ km/h

Downstream = $(x + y)$ km/h

ACQ,

Ritu can row downstream 20 km in 2h

and $\because d = s \times t$

$2(x + y) = 20$

$x + y = 10$ —(i)

also,

Ritu can row upstream 4 km in 2h.

$2(x - y) = 4$

$x - y = 2$ —(ii)

adding eq. (i) and (ii)

$2x = 12$

$x = 6$

putting this in eq. (i)

$6 + y = 10$

$y = 4$

→ Ritu speed in still water is 6 km/h

→ The speed of the current is 4 km/h.

ii) Let the no. of days taken by a woman and a man be x and y respectively

$\therefore$ work done by a woman in 1 day = $\frac{1}{x}$

work done by a man in 1 day = $\frac{1}{y}$

ACQ,

2 women and 5 men take 4 days to complete the work

i.e. they take 4 days to complete one work.

$\Rightarrow 4\left(\frac{2}{x} + \frac{5}{y}\right) = 1$

$\Rightarrow \frac{2}{x} + \frac{5}{y} = \frac{1}{4}$

also,

3 women and 6 men take 3 days to complete the work i.e.

they take 3 days to complete one work.

$\Rightarrow 3\left(\frac{3}{x} + \frac{6}{y}\right) = 1$

$\Rightarrow \frac{3}{x} + \frac{6}{y} = \frac{1}{3}$

Putting $\frac{1}{x} = p$ and $\frac{1}{y} = q$ in these eq.

we obtain.

$3p + 5q = \frac{1}{4}$

$\Rightarrow 8p + 2q = 1$

and

$3p + 6q = \frac{1}{3}$

$3p + 18q = 1$

by cross multiplication;

$\frac{p}{-20-(-18)} = \frac{q}{-3-(-8)} = \frac{1}{144-180}$

$\frac{p}{-2} = \frac{q}{-1} = \frac{1}{-36}$

$\frac{p}{-2} = \frac{-1}{36}$ and $\frac{q}{-1} = \frac{1}{36}$

$p = \frac{1}{18}$ and $q = \frac{1}{36}$

$p = \frac{1}{x} = \frac{1}{18}$ and $q = \frac{1}{y} = \frac{1}{36}$

$x = 18$, $y = 36$

∴ woman = 18 days

man = 36 days.

iii) Let the speed of train and bus be u km/h and v km/h respectively

ACQ,

It takes her 4 hours if she travels 60 km by bus and rest (i.e. 270 km) by train.

as,

$\text{time} = \frac{\text{distance}}{\text{speed}}$

$\frac{60}{u} + \frac{270}{v} = 4$ ----- (i)

also, if she travels 100 km by train and the remaining by bus, she takes 10 mins longer i.e. 4 hours and 10 mins.

$\frac{100}{u} + \frac{200}{v} = \frac{25}{6}$ ----- (ii)

Putting $\frac{1}{u} = p$ and $\frac{1}{v} = q$, in these eq

$60p + 270q = 4$ ---- (iii)

$100p + 200q = \frac{25}{6}$

$600p + 1200q = 25$ ----- (iv)

multiplying eq. (iii) by 10,

$600p + 2700q = 40$ — (v)

Subtracting (iv) from (v)

$1200q = 15$

$q = \frac{1}{80}$ ---- (vi)

substituting in eq. (iii)

$60p + 3 = 4$

$p = 1$

$p = \frac{1}{60}$

Date __/__/____

$P = \frac{1}{u} = \frac{1}{60}$ and $q = \frac{1}{v} = \frac{1}{80}$

$u = 60$ km/h and $v = 80$ km/h

Speed of train = 60 km/h

Speed of bus = 80 km/h.

1. i)

ii)

iii)